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THE GENERATION OF SOUND IN THE FLOW OF AN EXCITED GAS

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We examine the development of sonic perturbations in the bounded subsonic flow of a gas excited by oscillations.

1. In a number of practical problems it is necessary to have adequate uniformity in the parameters of the active gas flow. Thus, for example, in the utilization of thermodynamically nonequilibrium gas media in lasers the nonuniformity of the flux density may, to a great extent, determine the quality of the light bundle (brightness and angular dispersion). In the case of large Reynolds numbers, the flux is made turbulent, which leads to chaotic nonuniformities in the index of refraction for the medium. The effect of flux turbulization on the gradient dispersion has been investigated in detail in [1].

In the presence of V–T relaxation, the medium exhibits a second viscosity [2], which may exceed the magnitude of the first, and in the case of considerable nonequilibrium it becomes negative. The dissipation of the sound under these conditions in the case of stationary nonequilibrium has been studied in [3, 4]. The present paper is devoted to a study of the unique features involved in the development of sound perturbations in the flow of a gas excited by oscillations. The nonuniformity of the parameters of the medium in the direction of the flow leads to refraction of the wave and to the departure of the perturbations from the region of intensification. It is demonstrated that considerable growth in sound waves is possible only when turning points are present, and under certain specific conditions the flow is absolutely unstable with respect to the generation of the sound. Corresponding increments have been determined.

2. The flow of a gas excited by oscillation will be described by the equations of gasdynamics, by the equation of state, and by the equation for oscillation energy:

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho\mathbf{v}=0,\quad \rho\frac{\partial\mathbf{v}}{\partial t}+\rho(\mathbf{v}\nabla)\mathbf{v}=-\nabla P,$$

$$\frac{\partial S}{\partial t}+(\mathbf{v}\nabla)S=c_V\frac{T_0-T}{\tau T}+\frac{q}{T},\quad \frac{\partial\varepsilon_{os}}{\partial t}+(\mathbf{v}\nabla)\varepsilon_{os}=\frac{\varepsilon_{os0}-\varepsilon_{os}}{\tau}+n,$$

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$$\varepsilon_{os} + c_V T = \varepsilon_{os0} + c_V T_0, \quad P = \frac{R}{\mu} \rho T. \tag{1}$$

Relaxation to the equilibrium values of the temperature T_0 and the oscillation energy $\varepsilon_{os0}(T_0)$ is described in simplified terms by the single relaxation time τ, which is dependent both on the pressure and the temperature of the gas. We assume the gas to be polytropic, with heat capacity c_V. The effects associated with the thermal conductivity and frictional viscosity are assumed to be neglibily small.

Let us examine the flow of the gas through a rectangular tube with absolutely solid and thermally insulated walls. We assume the unperturbed flow to be uniform and steady. We might expect the simplified model (1) to describe the development of perturbations, for which the second viscosity considerably exceeds the first. We will set aside the questions of sound generation as a consequence of turbulence, these questions having been thoroughly studied in [5].

In the stationary case the system of equations (1) exhibits three integral motions and reduces to the following equation for the velocity of the flow:

$$\rho u = C_1, \quad P + \rho u^2 = C_2, \quad c_V T + \varepsilon_{os} + \frac{u^2}{2} + \frac{P}{\rho} - \int_0^x \frac{q+n}{u} dx = C_3,$$
$$(1 - M^2) \frac{du}{dx} = \frac{T_0 - T}{\gamma \tau T} + \frac{q}{c_p T}. \tag{2}$$

Throughout in the following we examine flows with $M < 1$.

The equations for the perturbations are derived through linearization of system (1). Two independent types of waves exist in the linear approximation, and namely, sound waves and entropy waves. All remaining results pertain to sound perturbations.

In the lateral cross section the perturbation is represented by standing waves. The boundary conditions are reduced to conventional quantization of the transverse components of the wave vector:

$$k_y = \frac{\pi}{L_y} l, \quad k_z = \frac{\pi}{L_z} m.$$

The relationship to the longitudinal coordinate x and to time for high-frequency sound ($\omega\tau \gg 1$) will be sought in the form

$$f = f(x) \exp[-i\omega t + i \int k_x dx + at].$$

Here f(x) is a smooth function $[(1/f)(df/dx) \ll k_x]$; ω, k_y, and k_z which do not change in the propagation of the wave.

The dispersion relationship and the link between the various quantities in the wave, accurate to terms of the first order of magnitude with respect to $1/\omega\tau \ll 1$, have a form that is usual for sound in a moving medium [6]:

$$(\omega - k_x u)^2 = k^2 c^2, \tag{3}$$

$$k^2 = k_x^2 + k_y^2 + k_z^2 = k_x^2 + k_\perp^2, \tag{4}$$

$$\tilde{\rho} = \frac{\tilde{P}}{c^2}, \tag{5}$$

$$\tilde{v}_i = \frac{k_i \tilde{P}}{\rho(\omega - k_x u)}, \tag{6}$$

$$\tilde{S} = 0, \tag{7}$$

$$\tilde{\varepsilon}_{os} = 0, \tag{8}$$

$$\tilde{P} = A \frac{kc}{\sqrt{Mk|v_{gr}|}} \exp\left[\int_0^x \left(\frac{N}{2} - a\right) \frac{dx}{v_{gr}}\right], \tag{9}$$

where $v_{gr} = \pm \frac{\sqrt{\omega^2 - k_\perp^2 (c^2 - u^2)}}{k}$ is the group velocity of the perturbations; $N = \left[\frac{\partial}{\partial P} \left(\frac{T_0 - T}{\tau T} + \frac{q}{c_V T}\right)\right]_{S,\varepsilon_h} P$. Relationships (3) and (4) describe the refraction of the sound being propagated at an angle to the direction of the flow, while relationships (5)-(9) describe the relationship of the various perturbation parameters and their change in the direction of the flow. In expression (9) for pressure the preexponential factor reflects the interaction of the wave with the flow, while the exponent represents the influence exerted on the V–T relaxation and release of heat applicable to the wave. The case of negative second viscosity corresponds to $N > 0$.

If the relaxation time exhibits the Landau–Teller form $P\tau \sim \exp(E/T)^{1/3}$, and the density of the heat liberation depends exclusively on temperature $q/c_V T = \Omega(T)$, then

$$N = \frac{T_0 - T}{\gamma \tau T} \left[1 + \frac{\gamma - 1}{3} \left(\frac{E}{T}\right)^{\frac{1}{3}}\right] - \frac{c^2 - c_0^2}{c^2 \tau} + \frac{\gamma - 1}{\gamma} \Omega \hat{\Omega}. \tag{10}$$

The continuity equation for the energy of the wave

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial x}(v_{gr} W) = NW. \tag{11}$$

The presence of a region within the flow of negative second viscosity ($N > 0$) leads to amplification of the sound. The greatest amplification occurs near the points of wave rotation, where $v_{gr} = 0$; $\omega^2 = k_\perp^2 (c^2 - u^2)$.

If the profile of $c^2 - u^2$ exhibits a maximum, it becomes possible to generate a sound with a frequency $\omega = k_\perp \sqrt{\max)c^2 - u^2)}$. From Eq. (11) we have the condition of generation:

$$N \geqslant \sqrt{-\frac{d^2 (c^2 - u^2)}{dx^2} \frac{1 - M^2}{2}}. \tag{12}$$

In the case of purely relaxation heating ($q = 0$) the function $c^2 - u^2$ exhibits a maximum in that section in which the velocity $u = c/\sqrt{\gamma + 2}$ is attained. In this case the increment is equal to

$$a = \frac{T_0 - T}{2\gamma\tau T} \left[\frac{\gamma - 1}{3} \left(\frac{E}{T}\right)^{\frac{1}{3}} + 1 - \sqrt{\gamma + 2}\right] - \frac{c^2 - c_0^2}{2\tau c^2}. \tag{13}$$

The flow is unstable with high nonequilibrium and where the relaxation time is sharply dependent on temperature. The existence of energy losses comparable to the relaxation heating, but without violating the monotonicity of the $u(x)$ profile, mitigates this condition. A sound is generated with the frequency $\omega = \sqrt{\gamma + 1}\, k_\perp u$ and $k_x = -k_\perp \sqrt{\gamma + 1}$.

If the maximum of $c^2 - u^2$ coincides with the maximum of the flow velocity and the heat losses are explicitly independent of x, condition (12) assumes the form

$$0 \leqslant a\tau = \frac{T_0 - T}{2\gamma T} \left[1 + \frac{\gamma - 1}{3} \left(\frac{E}{T}\right)^{\frac{1}{3}} - (\gamma - 1)\hat{\Omega} - \sqrt{\frac{(1 - (\gamma + 2) M^2)\left(1 + \frac{n}{q}\right) c_p T}{2M^2 (c_V + c_{0s})(T_0 - T)}}\right] - \frac{c^2 - c_0^2}{2c^2}, \tag{14}$$

where $(1 - (\gamma + 2)M^2)(1 + n/q) > 0$ is the condition for the maximum. Sound generation is possible if the Mach numbers are not excessively small and if the losses Ω are weakly dependent on temperature.

In either case, the developing perturbations occupy the entire region of the flow.

If the profile of the flow velocity (and, consequently, the profile of $c^2 - u^2$) exhibits a minimum, then a portion of the perturbations will be blocked between the two turning points. A somewhat unusual sound resonator is formed. The perturbations occupy a limited region of the flow and represent a discrete set of modes. The expressions for the frequencies and increments have the form:

$$\int_{x_{1i}}^{x_{2i}} \frac{\sqrt{\omega_i^2 - k_\perp^2 (c^2 - u^2)}}{c^2 - u^2} c dx = i + \frac{1}{2}, \tag{15}$$

$$\int_{x_{1i}}^{x_{2i}} \frac{(N - 2a_i)\omega_i dx}{\sqrt{\omega_i^2 - k_\perp^2 (c^2 - u^2)}(c^2 - u^2)} = 0. \tag{16}$$

The frequencies $\omega \gtrsim k_\perp \sqrt{\min(c^2 - u^2)}$; the increments $a_i \sim N_{av}/2$. The quasiclassical condition is $k_\perp u\tau \gg 1$.

3. These results are easily generalized to the case in which the relationship between the relaxation time and density and temperature differ from the Landau–Teller form. Let us also take note of the fact that although in the main we have been speaking of the relaxation heating of the gas, similar effects are possible with a different nature for the evolution of heating, with pronounced dependence on the parameters of the medium, e.g., in an electric discharge. These are described by relationships (1)-(12), (15), and (16) without a relaxation term.

In conclusion, we present estimates of the following parameters of oscillation excited gas. In the nonequilibrium case $(T_0 - T)/T \sim 0.1$, with relaxation times of $\tau \sim 10^{-3}$-10^{-4} sec, and with pressures of $P \sim 0.1$ atm, the second viscosity is negative and exceeds the first in the case of sound perturbations with a wavelength on the order of 1 cm. Under these conditions the increments amount to $\sim 10^2$ sec^{-1}.

NOTATION

ρ, T, P, S, $\mathbf{v}$, density, temperature, pressure, entropy, and velocity of the gas; ε_{os}, density of oscillation energy; q, heat generation, existing in addition to the relaxation heating; n, pumping; the x axis is directed along the channel; $u = v_x$; $\gamma = c_p/c_V$, adiabatic exponent; C_1, C_2, C_3, constants determined by the boundary conditions in the cross section x = 0; M = u/c, Mach number; $c = \sqrt{\gamma P/\rho}$, velocity of the high-frequency sound; ℓ, m, whole numbers; L_y and L_z are dimensions of the channel along the y and z axes, respectively; k_x, longitudinal component of the wave vector; ω, frequency; a, perturbation growth increments; A, amplitude; $c_0 = \sqrt{\frac{c_{os} + c_p}{c_{os} + c_V} \frac{P}{\rho}}$, velocity of the low-frequency sound; $c_{os}(T_0) = d\varepsilon_{os0}/dT_0$, heat capacity of the oscillation degrees of freedom; $\hat{\Omega} = \frac{d\Omega}{dT} \cdot \frac{T}{\Omega}$; $i \gg 1$, mode number; x_{1i} and x_{2i}, corresponding points of rotation; $W = |\tilde{P}|^2 \omega/\rho k c^3$, wave energy; V–T, oscillation-translation relaxation.

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